thought exercise

ANALYZING ALGORITHM

Our Analyzing Algorithm Thought Exercise encourages the careful analysis of the disciplined thinking of a mathematician, that is, a mathematician whose assertion is a coherent composition of mathematically justifiable statements that help to substantiate the reason that a purported solution is, at the very least, reasonable. Engaging learners in this way underscores the importance of proof—truth or credibility established by sound reasoning—in mathematics as well as the importance of being able to clearly communicate or present proof in order for it to be accessible to one's audience.

During this Thought Exercise, begin by allowing learners to critically observe the algorithm for about one minute. They should be attempting to make sense of the mathematician's thinking as well as recognizing properties, postulates, and/or theorems that justify the mathematicians' choices. A teacher may introduce this Thought Exercise with fully completed algorithms, where every line from start to the final answer (or simplest form of the expression) is shown. This is illustrated below in Figure A. After learners have had an opportunity to observe, press learners to read and fully understand the original prompt. In the case of an expression or equation with multiple terms, for example, a teacher can ask learners to identify the terms of the original expression or equation. The teacher or a learner would box, circle, or in some other way distinguish the terms. Then, the teacher could ask learners to compare the original expression to the first line of the algorithm in order to determine what has occurred with respect to each term to create the new, equivalent expression. Learners should describe the thinking of the mathematician and justify the thinking with specific properties, postulates, and/or theorems. This comparison of the previous line to the line under investigation continues until the learners have reached the end of the algorithm. At the conclusion of this work, learners will have determined whether the mathematician has provided sound reasoning to support his/her final assertion.

$= 47(50 + 5 + 1)$	GIVEN
$= 47(50) + 47(5) + 47(1)$	DISTRIBUTION
$= 47(50) + 47(10 \times \frac{1}{2}) + 47$	SUBSTITUTION
$= 47(50) + (47 \times 10)\frac{1}{2} + 47$	MULT. ASSOCIATION
$= 47(50) + (470)\frac{1}{2} + 47$	SUBSTITUTION
$= 47(50) + [(400)\frac{1}{2} + (70)\frac{1}{2}] + 47$	DISTRIBUTION
$= 47(50) + 235 + 47$	SUBSTITUTION
$= 2350 + 235 + 47$	SUBSTITUTION
$= 2350 + 232 + 3 + 47$	SUBSTITUTION
$= 2350 + 232 + (3 + 47)$	ADD. ASSOCIATION
$= 2350 + 232 + 50$	SUBSTITUTION
$= 2350 + 50 + 232$	ADD. ASSOCIATION
$= 2400 + 232$	SUBSTITUTION
$= 2632$	SUBSTITUTION

Context of Instructional Design

This Thought Exercise was created for Red Band, a group of 2nd and 3rd graders in their first year of studying with us. The particular algorithm given here was strategically presented to learners to have them notice the connection between additive and multiplicative association. As Red Band worked through this Thought Exercise, they realized how often substitution is used within an Analyzing Algorithm TE, but in each situation they were pressed to consider the specific substitutions made which enabled the mathematician to carry out more elegant or efficient work.

For example, in the ninth line of the algorithm, the mathematician substituted 235 for 232 and 3 because they were thinking critically about what would combine efficiently with the 2350 and saw that 47 was only 3 away from 50.

the number lab