thenumberlab

The Concept Study

The **Concept Study** is the segment of the Math Block that is devoted to constructing understanding of a mathematical concept or idea. Thus, Concept Studies are thoughtfully prepared, focused community discussions. The architecture of a Concept Study is typically a series of expressions combined with very intentional questions that are meant to lead learners from a mathematical understanding with which they are quite confident to new mathematical terrain or perhaps a different vantage point by which to survey terrain previously experienced. In other words, being guided through a purposefully formulated series of prompts, learners make cognitive leaps that ultimately result in new or deeper math knowledge. This is our way of pulling on the aforementioned threads - the mathematical ideas and knowledge that learners have attained - and then binding them together for learners to fully grasp concepts. Figures 1 and 2 on the following pages are examples of the ways in which Concept Studies can be structured in order to meet the aim of building from or extending learners' knowledge.

> "Concept Studies are thoughtfully prepared, focused community discussions

The Concept Study illustrated in Figure 1 shows a teacher's intent to engage learners in a discussion concerning addition. For the purpose of this discussion, the teacher has defined addition as combining like terms. Defining addition in this way, the teacher has in essence conceptualized addition.

Learners are being pressed to recognize addition as an idea rather than a mere operation: something that one does, a function, or a purely algorithmic process. In order to help learners construct addition as the concept of combining like terms, the teacher provides a series of cues or prompts. Each prompt is an additive expression containing terms that are presented in common fraction form, standard decimal form, or algebraically.

Figure 1

Concept Study

Concept + Connective Ideas + Aims	Addition: Combining Like Terms • Learners will solidify their understanding that addition is the combining of terms - expressions of specific counts - that are alike or of the same count. They will know this as a truth regardless of the number forms or algebraic expressions that serve as terms in an additive expression. In an additive series, learners will recognize the usefulness of the associative property of addition as well as the principle of substitution for creating like terms.

Prompts	Cues and/or Understandings
$8\frac{3}{4} + 11\frac{2}{4}$	The associative property of addition allows for the combining of like terms. $(8 + 11) + (\frac{3}{4} + \frac{2}{4})$
$9\frac{5}{9} + 11\frac{2}{3}$	Substitute to create like terms. $9\frac{5}{9} + 11\frac{2}{3} = 9\frac{5}{9} + 11\frac{6}{9}$, as $\frac{1}{3} = \frac{3}{9}$; thus: $(9 + 11) + (\frac{5}{9} + \frac{6}{9})$
5.23 + 9.4	Substitute to create like terms. $5\frac{23}{100} + 9\frac{4}{10} = 5\frac{23}{100} + 9\frac{4}{10}$, as $\frac{1}{10} = \frac{10}{100}$; thus: $(5+9) + (\frac{23}{10} + \frac{40}{100})$
$-7c + 4d + -6c$	$(-7c + -6c) + 4d$
$2 + \frac{3}{m}$	Substitute to create like terms. $2(1) + 3(\frac{1}{m}) = 2(m(\frac{1}{m})) + 3(\frac{1}{m}) = 2m(\frac{1}{m}) + 3(\frac{1}{m}) = (2m + 3)(\frac{1}{m})$

By varying the number forms in the expressions, it is clear that the teacher is attempting to press the learners to leverage ideas that are applicable regardless of the number forms. In other words, the learners should realize that the number forms are inconsequential with regard to the actual concept of addition. The ideas or understandings that the teacher seeks to help learners build upon to conceptualize addition as combining like terms are: additive association - changing the order by which terms are combined in an additive series - and substitution principle - if the value of *a* is equivalent to the value of *b*, then *b* can replace or be substituted for *a* in an expression.

Applying these understandings, learners read expressions with the intent of identifying the terms, determining whether there are terms that are alike, associating to combine the like terms, and/or substituting a term for a specific equivalent in order to create like terms. In this way, addition expressions are actually problems or situations that require learners to think as opposed to rotely applying a specific algorithm.

Figure 2 is an illustration of a Concept Study in which a teacher attempts to help learners leverage certain understandings related to division in order for the learners to divide with polynomials. With learners already understanding $^{13}/_{12}$, for example, as equivalent to $1\,^{1}/_{12}$ by the following logic:

- $^{13}/_{12} = 13(^{1}/_{12})$ → **definition of division**
- $13(^{1}/_{12}) = (12 + 1)(^{1}/_{12})$ → **substitution**

Figure 2

Concept Study

Concept + Connective Ideas + Aims	Division with Polynomials: An Algebraic Representation of Division - Part C • Definition of division • Substitution • Multiplicative identity • Multiplicative inverse(reciprocal) Building on their understanding of division as a multiplicative idea along with substitutive thinking, learners will expand their understanding of division through **dividing a polynomial by a binomial**.

Prompts	Cues and/or Understandings
$\frac{6x+18}{3}$ *and* $\frac{x+7}{x+5}$	Ask learners to observe similarities and differences between the two expressions. The observations should lead learners to recognize that while $\frac{6x+18}{3} = \frac{6x}{3} + \frac{18}{3}$, $\frac{x+7}{x+5}$ does not equal $\frac{x}{x} + \frac{7}{5}$ *nor* $\frac{x+7}{x} + \frac{x+7}{5}$. Use substitution property to prove. Learner should understand that the unit being counted is: $\frac{1}{x+5}$, and its numeration is the quantity of x and 7.
$\frac{x+7}{x+5}$	Because 1 = $\frac{x+5}{x+5}$, then $\frac{x+5}{x+5} + \frac{2}{x+5}$
$\frac{x+18}{x+5}$	$= \frac{x+5}{x+5} + \frac{13}{x+5} = 1 + \frac{13}{x+5}$
$\frac{3x+21}{x+5}$	$= 3(\frac{x+3}{x+5}) + \frac{12}{x+5} = 3 + \frac{12}{x+5}$ $\frac{3x^2+33}{x+5}$
$\frac{5x+27}{x+5}$	$= 5(\frac{x+3}{x+5}) + \frac{12}{x+5} = 5 + \frac{12}{x+5}$ $\frac{5x^2+15}{x+5}$

- $(12 + 1)(^1/_{12}) = 12(^1/_{12}) + 1(^1/_{12}) \rightarrow$ **distribution**
- $12(^1/_{12}) + 1(^1/_{12}) = 1^1/_{12} \rightarrow$ **substitution**

Learners will use the various prompts to connect and transfer these understandings to expressions such as, $^{x+7}/_{x+5}$, an expression also included in Figure 2. Thus, learners will know, by the same logic or pulling on the same threads, that:

- $^{x+7}/_{x+5} = x+7\ (^1/_{x+5}) \rightarrow$ **definition of division**
- $x+7\ (^1/_{x+5}) = ((x + 5) + 2)(^1/_{x+5}) \rightarrow$ **substitution and additive association**
- $((x + 5) + 2)(^1/_{x+5}) = {}^{x+5}/_{x+5} + {}^2/_{x+5} \rightarrow$ **distribution**
- $^{x+5}/_{x+5} + {}^2/_{x+5} = 1 + {}^2/_{x+5} \rightarrow$ **substitution**

During Concept Studies, as in every aspect of the Math Block, questions provide scaffolding; the teacher does not "show" or "tell" learners what they must do. And not Concept Studies facilitate discourse intended to create new knowledge, understanding, and experience, building upon the prior knowledge, understanding, and experience of those in the conversation.

When our learners are engaged in discourse, they formulate, communicate, and critically assess arguments. We invite, encourage, and coach learners to communicate and grapple with conflicting claims and counter arguments. *In this way, we normalize the messiness of learning.* We recognize that struggle is indeed intrinsic to learning; thus, we embrace struggle. And being a mediator of the discourse, the teacher may ask the band whether they agree with the argument. If anyone disagrees, the teacher or a member of the band will ask for evidence or reasoning that supports the counterclaim. In this way, we normalize the messiness of learning. We recognize that struggle is indeed intrinsic to learning; thus, we embrace struggle.